Intermittency and Multifractal Velocimetry in Rayleigh-Taylor and Convective, Richtmyer-Meshkov Flows

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The stability of interfaces between two superposed fluids of different density were first studied by Lord Rayleigh and Taylor (1950) for the case when the dense fluid is accelerated towards the less dense fluid, Chandrasekhar (1961). The objective of this study is the comparison of models and experiments that model adequately describing mixing and subsequent evolution of Rayleigh-Taylor instability (RT) and Richtmyer-Meshkov (RM) instability.
We present applications of Fractal analysis in Shock induced compressible flows as well as in fractal grid wakes in order to compare different measures of non-homogeneous flows that aid the understanding of complex mixing processes. We concentrate on the applications of advanced multi-fractal methods in order to evaluate the scale to scale transfer of energy and other descriptors of great importance in mixing processes. In particular we discuss the evolution of fluxes as molecular mixing takes place, here the use of fast reactive indicators such as Phenoftalein, provides multi-fractal analysis and improvements on Structure function calculations on standard PIV, and on several methods used in experimental fluids mechanics, calibrated towards the understanding of molecular mixing and the role of vorticity and helicity in the analysis of velocity vectors, the divergence and vorticity-stream function parameter spaces in stirring and mixing in terms of the Atwood number:

\[ A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \]
growthrate to occur at a finite wavelength. For the viscous two-layer case, where the upper layer (density $\rho_1$) is denser than the lower layer (density $\rho_2$), the wavelength $\lambda_m$ of maximum growthrate is

$$\lambda_m \approx 4\pi \left( \frac{v^2 (\rho_1 + \rho_2)}{g (\rho_1 - \rho_2)} \right)^{\frac{1}{3}}$$

where $v$ is the mean kinematic viscosity of the two layers and $g$ is the acceleration of gravity. The corresponding maximum growthrate is

$$n_m \approx \left( \frac{2g \pi (\rho_1 - \rho_2)}{\lambda_m (\rho_1 + \rho_2)} \right)^{\frac{1}{2}}$$

While the linear theory for two infinite layers is well established, the development of the instability to finite amplitude is not amenable to analytic treatment. There have been a number of semi-analytical and numerical studies in recent years, but they all involve simplifying assumptions which raise serious doubts about their validity particularly when applications to mixing are sought. An overview of the subject by Sharp(1984) characterized the development
The advance of a Rayleigh-Taylor front is described in Linden & Redondo (1991), and may be shown to follow

\[ d = 2 \ c A t^2 \]

where \( d \) is the width of the growing region of instability, \( g \) is the gravitational acceleration and \( A \) is the Atwood number.
The corresponding maximum growth rate has been described in Redondo and Linden (1990) and Linden and Redondo (1991), where the thickness grows quadratically in time. The proportionality factor $c$ is considered to be a (supposedly universal) constant, although some dependence with the Atwood number and the initial conditions of the plate removal or random numerical fluctuations is expected (Castilla and Redondo 1994).
Experimental Setups

Atwood No. = 0.002-0.1
Description of the RT experiments

The experiments consisted of a release of a dense fluid in a rectangular perspex tank of height $H$ 0.50 m, length $L$ 0.40 m and width $W$ 0.20 m. The two fluids are initially separated by a removable stainless steel sheet, 1.5 mm thick, in the centre of the tank. Fresh water is placed in the lower half of the tank and the sheet, sliding in tight fitting grooves, is pushed across and sealed with silicone grease, and finally the dense layer of brine is placed on top. The experiment is initiated by withdrawing the plate horizontally through a seal in the end wall of the tank. A 0.6 W argon laser light passing through an cylindrical lense was used to produce a 2 mm thick sheet of high intensity light. Fluorescein dye was added to one of the layers, and this produces a brilliant green image in the laser light. The leading edge of the advancing front was demarcated by this technique, and images of the small scale structures were obtained in the experiments described in Redondo and Linden (1993). Further analysis allowed for a large range of intensity values to be analysed and not just the Volume fraction isoline of 50%.

The fluorescein is only added in very small quantities so that it acts as a passive tracer. Using suitable orientations of the light sheet, side and end elevations and plan views of the flow were obtained. The second method of flow visualization which was used involved the use of a pH indicator to mark the flow where molecular mixing occurs. As discussed in Linden y Redondo (1991), it was necessary to have the Atwood number greater than 0.005 to ensure that the plate did not affect significantly the RT front. The perturbations induced by the plate were reduced using the method described in Dalziel(1994).
Molecular mixing in Rayleigh–Taylor instability. Part I: Global mixing

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This paper describes experiments on the mixing produced by Rayleigh–Taylor instability between two miscible fluids. A layer of brine is placed over a layer of fresh water in a gravitational field, and the ensuing flow is visualized by laser-induced fluorescence and measurements are made of the concentration fields as the flow develops. It is found that large-scale disturbances develop which produce intermingling of the fluids, but molecular mixing occurs as a result of small-scale instabilities which grow on the edges of the larger scale motions. When the system overturns stably stratified fluid is produced, and the mixing efficiency of the process is measured and found to be large compared with other forms of mixing. It is suggested that this increased efficiency is due to the fact that much of the mixing occurs when the system is unstable.

I. INTRODUCTION

Rayleigh–Taylor (RT) instability occurs at an interface between dense fluid and less dense fluid when the latter is at a higher pressure. The most familiar example is that of heavy fluid lying on top of light fluid in a gravitational field. Another example is the case of laser-induced implosion of glass encapsulated deuterium targets. These targets are spherical, and, during the implosion process, high pressures develop in the less dense deuterium as the glass shell decelerates and RT instability ensues.

In many practical circumstances it is important to determine the amount of mixing that occurs between the fluids as a result of the RT instability. An important component of the overall mixing in stratified fluids is due to overturning processes, such as breaking internal waves or Kelvin–Helmholtz billows, in which local density inversions are produced. For atmospheric and oceanic flow estimates of the vertical mixing are required in order to parametrize these mixing processes in numerical models.

In this paper we present an investigation on the mixing produced by a turbulent, unstably stratified flow. The aim of the research is to determine the amount of molecular mixing that occurs during the RT instability process using laboratory experiments. The modeling approach adopted here is to use two miscible liquids, with the dense liquid accelerated by gravity toward the less dense liquid. The properties of the mixing process are examined using a range of laboratory techniques as outlined below.

The stability of an interface between two superposed fluids of different density was studied by Rayleigh, and Taylor carried out a linear stability analysis and noted that than the lower layer (density \( \rho_2 \)), the wavelength \( \lambda_m \) of maximum growth rate is

\[
\lambda_m \approx 4\pi \left[ \frac{\nu}{g (\rho_1 + \rho_2) / g (\rho_1 - \rho_2)} \right]^{1/2},
\]

where \( \nu \) is the mean kinematic viscosity of the two layers and \( g \) is the acceleration of gravity. The corresponding maximum growth rate is

\[
n_m \approx \left[ \frac{\pi (\rho_1 - \rho_2)}{\lambda_m (\rho_1 + \rho_2)} \right]^{1/2}.
\]

While the linear theory for two infinite layers is well established, the development of the instability to finite amplitude is not amenable to analytic treatment. There have been a number of semianalytical and numerical studies in recent years, but they all involve simplifying assumptions which raise serious doubts about their validity, particularly when applications to mixing are sought.

A discussion of the subject by Youngs characterized the development of the instability through three stages before breaking up into chaotic turbulent mixing. Initially, a perturbation of wavelength \( \lambda_m \) grows exponentially with growth rate \( n_m \). When this perturbation reaches a height of approximately \( \frac{1}{2} \lambda_m \), the growth rate decreases and larger structures appear. In the final stage the scale of dominant structures continues to increase and memory of the initial conditions is lost; viscosity does not appear to affect the growth of the large structures.

This latter result concerning the independence of the large amplitude structures on the initial conditions leads to the result that the width of the mixing region, which develops between the two layers, depends only on \( \rho_1, \rho_2, g \), and time \( t \). Then dimensional analysis gives

\[
\delta = 2cg \left[ (\rho_1 - \rho_2) / (\rho_1 + \rho_2) \right] t^4,
\]
In experiments where plan view laser-induced fluorescence (LIF) was used to investigate the horizontal structure of the RT instability, three-dimensional blobs could be seen to grow despite the fact that the initial perturbations produced by the withdrawal of the plate were two dimensional. This observation provides additional confirmation that the growth of the mixing region and the nature of the observed disturbances appear to be independent of the initial conditions.

Superimposed on these wave motions are smaller-scale motions characteristic of mixing in stratified fluids. These motions eventually decay in the manner described by Pearson and Linden.\(^\text{15}\)

**B. Quantitative results**

1. **The thickness of the mixing region**

   Measurements of the half-width \(\delta\) of the mixing region were taken from the flow visualization experiments. Both shadowgraphs, in which both edges of the mixing region were visible, and dye visualizations, in which only one edge was visible were used, and measurements were restricted to the central portion of the tank where the region was fairly uniform. The values of \(\delta\) then correspond to averages over the horizontal area of the interface.

   An example for one experiment is shown in Fig. 1. In accordance with (3) the data exhibit a quadratic time dependence on time \(t\). A least-squares fit to the data with \((\delta/H)^{1/2}\) plotted against \(t/T\) is shown, and from this we see that the line does not pass through the origin. The mixing region has a finite initial thickness \(\delta_0\) that corresponds to the disturbances generated by the removal of the plate. In the example shown in Fig. 1, \(\delta_0 = 0.8\) cm, which is consistent with the observed size of the vortices shed by the plate. This distance is small compared with the depth of the tank, and so may be neglected during the later growth of the mixing region, as suggested by the qualitative observations described above. Once the initial conditions are forgotten, the rate of growth of the mixing region depends only on \(\delta, A,\) and \(g\) (assuming molecular effects are negligible). Dimensional analysis then gives

   \[
   \frac{d\delta}{dt} = 2(c_g A \delta)^{1/2},
   \]

   and integration gives

   \[
   S^{1/2} = \delta_0^{1/2} + (2c_g A t^2)^{1/2},
   \]

   where the initial condition \(\delta = \delta_0\) at \(t = 0\) has been used. When the mixing region is thick \(\delta \gg \delta_0\), (6) becomes \(\delta = 2c_g A t^2\), which recovers (3).

   Values of \(c\) are determined from the slopes of plots such as shown in Fig. 1. A total of 49 experiments were carried out and the values of \(c\) are plotted against the Atwood number \(A\) in Fig. 2. At low values of \(A\), the values of \(c\) are somewhat scattered due to the effects of the plate removal, but at higher values of \(A\), \(c\) is approximately constant and takes the value 0.06.
Molecular mixing in Rayleigh–Taylor instability

\( \tau = 0.5 \)

\( \tau = 1.0 \)

\( \tau = 1.5 \)

\( \tau = 2.0 \)
Experimental Visualizations
Ph phenolphthalein color change indicator
Measurements on the Advance of a RT Front

P. F. Linden, J. M. Redondo and D. L. Youngs

\[
\left( \frac{h_1}{H} \right)^{1/3}
\]

\[
\tau - \tau_0
\]
LES of RT flow (Garzon et al 2004)
The Evolution of the Instability
Velocity - Vorticity - Density
The Evolution of the Velocity Field

$A = 1 \times 10^{-4} - 128^2$ elements mesh
RT front at $t/T = 1, 2$ and 3
Pearlescence Tracer Transport
LIF with Passive Tracers

- Chemically benign tracer
- Chemically reactive tracer
The turbulent mixing of stratified fluids is of concern in many fields of both numerical and experimental investigations of mixing processes in an environment with an stable interface lying beneath a Rayleigh-Taylor unstable one.

Here we seek not only to quantify the growth of the R-T instability on the unstable interface, but to characterise the mixing it induces across the (locally) stable interface.
RT instability growth \((z,t)\)

Lawry & Dalziel (2010)
The global mixing efficiency of the overall process is measured. The behaviour of individual plumes is very different to the environmental energy, which is generated by an oscillating grid at the bottom of the tankself-similarity structure flow and relate them to mixing and stirring. The evolution of the mixing fronts are compared and the topological changes merging of the plumes and jets are examined for different configurations.
RM driven fronts (Low A)

Experimental and numerical results on the advance of a mixing or non-mixing front occurring at a density interface due to gravitational acceleration are analyzed considering the fractal and spectral structure of the front. The experimental configurations presented consists on an unstable two layer a dropping box on rails and shock tube high Mach number impulse across a density interface air/SF6 in the case of Richtmyer-Meshkov instability driven fronts.
Experimental and numerical results on the advance of a mixing or non-mixing front occurring at a density interface due to a sudden acceleration shock have been analyzed considering the fractal structure of the fronts as well as several geometrical indicators of the local mixing processes. The experimental configurations compared are several and have been previously described in Castilla and Redondo (1993) for RM fronts. This later configuration, that has also been employed by Jacobs et al. consists on a free falling box, where previously a stable sharp density interface has been formed using different fluid combinations, alcohol water, oil, mercury, air. The initial density difference is characterized by the Atwood number. The evolution of the Richmeyer-Meshkov instability is non dimensionalized also by $\tau$. As the free falling box is suddenly stopped, an upwards acceleration, generates a combination of sharp spikes and bubbles, which reach a maximum, function of the Atwood number, and the mixing efficiency at the front. A similarity solution may be found, following Youngs(1991). The quantitative results obtained by image analysis of the front evolution described by Castilla and Redondo (1993) have been explained and compared with numerical simulations, the velocity structure near the front shows different scaling properties between density or volume fraction, velocity and vorticity found by Redondo and Garzon(2004).
Figure: evolution of the structure of the RM front at times $T=0-43$ (From Jacobs IWPCTM6 1997).
\[
u' = \frac{1}{10} \sqrt{\delta c g A} \quad \epsilon = \frac{u'^3}{\lambda_m} \quad \eta = \left( \frac{v^3}{\epsilon} \right)^{\frac{1}{4}}
\]

\[
\delta = 0.14 \Delta V A \ t
\]

\[
\delta = c \Delta V A \ t^{\frac{2}{3}}
\]

giving a reasonable fit to some of the slowest growth rates in the experiments, but the restoring effect of gravity, which we have to remember, that is always present in the RM drop experiments prevents further growth.

A Damped oscillation of the interface sets in after the shock decays. Slower front growths of the interface were recorded mostly in the Hg - H_2 O experiments.
The reasons for the intermittency study are: 1) The fact that large scales are most likely to affect the front interfaces, which may be due to wall or experimental asymmetries. 2) The easy identification of the integral length scale of the turbulence from velocity data or visual analysis. 3) The effect of the stratification on scales larger than the Buoyancy or the Ozmidov scale. 4) The difficulty in resolving scales near the Kolmogorov length scale, which are generally supposed not to be greatly affected by stratification. Redondo (1990) found that the stable stratification reduces the fractal dimension of the turbulence. There is a double influence of the stratification on a turbulent density interface, first to reduce the overall vertical displacements by means of an increased transfer of kinetic to potential energy, and from vertical to horizontal length scales.

More detailed experiments will be needed to be able to determine the exact mechanisms which reduce the effective fractal dimension, as well as the effect of higher order geometrical parameters, such as the structure functions, which seem to be relevant in non-homogeneous fluids (Mahjoub et al 1998).

The structure of a blob shows a relatively sharp head with most of the mixing taking place at the sides due to what seems to be shear instability very similar to the Kelvin-Helmholtz instabilities, but with sideways accelerations. The formation of the blobs with their secondary instabilities produces a turbulent cascade, evident just after about 1 non-dimensional time unit, from a virtual time origin that takes into account the linear growth phase, as can be seen by the growth of the fractal dimension in the front in time.
Effect of initial conditions
Fractal Dimension

\[ \text{nboxes(size)} = \left( \frac{1}{\text{size}} \right)^{\text{fractal dimension}} \]
Scaling Laws in Geophysical Flows

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Abstract. Statistical analysis of velocity structure functions are presented for turbulent flows at 1m above the bottom in a shallow (2m) bay in Denmark. High frequency (25 Hz) time series were collected mid day of 9 and 12, September 1997. The turbulent flow on day 12 was more energetic than on day 9 as a result of strong wind waves. The absolute scaling exponent $\xi_p$ was shown to have scale dependent behavior. In contrast, the relative scaling exponents $\zeta_p$, calculated using the extended self similarity (FSS) method and calculating the third order structure function with the modulus, was found to have a scale-independent behavior and deviated from the Kolmogorov K41 law. Moreover, values of $\zeta_p$ on day 12 were more intermittent than on day 9. This result indicates the influence of the wind and waves on the scaling laws of the velocity structure functions.

1 Introduction

The statistical and scaling properties of geophysical flows have recently received attention because of their intrinsic physical interest. Many efforts (Kolmogorov (1941), Kolmogorov (1962), Frisch et al. (1978) and Anselmet et al. (1984)) have been devoted to the study of turbulence phenomenon including theoretical, numerical, and experimental observations, all using velocity structure functions, which are the best indicators of the intermittent character of turbulence. Most of these works

The present work is a detailed investigation of turbulence using velocity data collected in a shallow, semi enclosed bay in Denmark. We analyze the absolute scaling exponent $\xi_p$ and the relative scaling exponent $\zeta_p$, where $\zeta_p$ is obtained using the Extended Self Similarity (ESS) property (Beuzi et al. (1993)). In-situ measurement of velocity were made on 9 and 12 September, 1997, at a height of 1 m above the sand bottom of the shallow (2 m mean depth) littoral zone in Knebel Vig bay. Knebel Vig is located at 56.20° N, 10.50° E and is a small O(3 km), shallow (maximum depth 18 m) bay that is connected by a narrow channel to the larger Aarhus Bight. Ten minute time series of velocity $\langle u, v, w \rangle$, acoustic Backscatter, and tilt and compass angles were sampled at 25 Hz using acoustic Doppler velocimeter (ADV model Ocean, Sontek Inc.). The probe resolved velocities as low as 0.25 cm $s^{-1}$ and was accurate to 0.4 cm $s^{-1}$ and 0.1 cm $s^{-1}$ for horizontal and vertical components, respectively. The probe was mounted in a “down-looking” mode on a metal tripod and data were collected in real-time. Prior to analysis, spurious velocities were identified by low correlations (< 70%) and signal strengths (< 5 db) between the 3 transducer channels and edited from files. Scaling exponents are calculated for two different days, for measurement at height of 1m above the bottom. The influence of the large scale non-homogeneity of waves due to wind on the scaling properties of the longitudinal velocity structure functions is investigated.
Structure Functions in Complex Flows

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Abstract. The turbulence in the ocean and atmosphere is most of the time non-homogeneous in nature. These spatial changes could affect the structure of the turbulence. In this work a classification is proposed to determine the intermittency and mixing ability. The variation of the structure functions and the scaling exponent in decaying non-homogeneous turbulence produced by a grid and by a jet is measured with a sonic velocimeter SONEK3-D. We use Extended Self Similarity (ESS) to obtain better estimates of the scaling exponents of the structure functions of order up to the 6th. We study the variation of the absolute scaling exponents $\zeta_p$ and relative scaling exponents $\xi_p$ as a function of distance from the source of turbulence. In most cases, the absolute scaling exponent $\zeta_3$ is shown to vary as function of the separation distance $l$. On the other hand the relative scaling exponents $\xi_p$ depend on the location of the flow and in most cases the deviations from the Kolmogorov 1941 scaling are related to the intermittency.

Key words: intermittency, scaling laws, non-homogeneous turbulence.

1. Introduction

Kolmogorov's (1941) local similarity theory K41 [1–4] assumed that in fully developed turbulence all statistically averaged quantities at scale $l$ depend only on the mean dissipation rate $\langle \varepsilon \rangle$ and $l$, where $l$ lies within the inertial sub-range. The velocity differences $\delta u_l$ across the distance $l$ have a power-law dependence on $l$ and the scaling exponents of the $p$th-order moment of velocity increments is given by

$$\langle \delta u_l^p \rangle \sim \langle \varepsilon \rangle^{p/3} l^{p/3}. \quad (1)$$

Experimental [1] and numerical [5] results found that $\langle \delta u_l^p \rangle$ deviates from the $p/3$ law. This has been referred to as the intermittency correction to K41. This correction takes into account the intermittent behaviour of the energy dissipation averaged over a ball of size $l$. $\varepsilon_l$. Assuming $\langle \varepsilon_l^{p/3} \rangle \sim l^{p/3}$, the $p$th-order moment of velocity increment is then defined as

$$\langle \delta u_l^p \rangle \sim l^{p/3}. \quad (2)$$
Remarks and Conclusions

The traditional $\delta_1=cl \, Ag \, t^2$ model for the growth rate of the RT instability is not an adequate description for flows where the initial conditions are in some sense inhomogeneous. While it is clear that a quadratic component remains, there is a spatial dependence in this component in addition to a linear term resulting from the vortex sheet. In these experiments the vortex-driven flow down the right-hand is dominated by a linear growth whereas the flow elsewhere follows the quadratic law much more closely. We therefore recommend that comparisons should not be limited to simply the $c_1$ constant, but should encompass more details of the growth and internal structure of the flow. The absence of a homogeneous quadratic growth suggests that the flow is not fully self-similar over all scales. However, the existence of a $k^{-5/3}$ concentration fluctuation power spectrum and constant fractal dimension indicates that internal similarity is still attained. As is to be expected, this structure changes significantly once the mixing region reaches the top and bottom of the flow domain.
Conclusions

Molecular mixing in the early stages of RT has been directly visualised with LIF

- Arguments based on growth rate used to justify accuracy of Front-Tracking schemes are shown to be inaccurate
- A taxonomy of changes in the equilibrium (or not) cascade may lead to more physically realistic (and understandable) models to parameterize sub-grid scaling
- Multi-fractal geometry we can also establish certain regions of higher local activity used to establish the geometry of the turbulence mixing
- Care has to be taken when interpreting the direct 3D Kolmogorov cascade and the Inverse 2D Kraichnan Cascade.
- It is very interesting to use ESS and the third order structure functions to investigate the scale to scale transfer of energy
BIBLIOGRAPHY


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Further Work

- Turbulent Diffusion over unstable and convective stable density interfaces with PIV
- Isolated RT and RM and ring impingement on stable density interfaces
- Influence of interfacial shear on turbulent diffusion
- Reactive and time delay mixing interactions

- Estimates of mixing efficiency